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1981 J. Phys. A: Math. Gen. 14 L433

(http://iopscience.iop.org/0305-4470/14/11/003)

J. Phys. A: Math. Gen. 14 (1981) L433–L435. Printed in Great Britain

LETTER TO THE EDITOR

Once more on multi-dimensional time

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Received 27 August 1981

Abstract. Conditions are briefly discussed which a transformation in a symmetrical six-dimensional space–time should fulfil.

Among generalisations of the ordinary four-dimensional space–time the symmetrical six-dimensional space–time attracts particular attention. It may be of help in understanding the asymmetry of space and time and the light barrier and may even play a role in the theory of electroweak interactions. At present the scheme of Cole (1980) seems to be most promising. If in it the time parts of all six-vectors are made parallel, the equations of special relativity are recovered. In a recent letter conditions were briefly discussed which would warrant this parallelism (Strnad 1980). It was proposed that the principles of special relativity, including the reciprocity condition for inertial frames, generalised to six dimensions, may suffice in this respect. In a comment this statement was criticised by Cole (1981). This short contribution intends to analyse and clarify the idea criticised.

The main point at issue is this. If, on the one hand, the reciprocity condition is not taken into account, Cole's scheme seems to be incomplete in the sense that its equations alone do not lead to a transformation between inertial reference frames in explicit form. In fact, no such transformation has yet been given. On the other hand, if the reciprocity condition is used it is difficult to avoid the conclusion of the Letter. In the following this is shown using a counterexample which is based on the derivation of the general Lorentz transformation in four dimensions. The reciprocity condition in the form 'if v is the velocity of origin O' of frame S' as measured in frame S, then $-v$ is the velocity of origin O of frame S as measured in frame S'' is a necessary ingredient of this derivation (Pars 1921). In the first part of the derivation the colinear Lorentz transformation is obtained between two frames having mutually parallel space axes and the relative velocity lies in the direction of a common space axis. Only after two subsequent rotations in the space of one frame with respect to the other bring us to the general Lorentz transformation in four dimensions. Now the relative velocity is inclined to the space axes, while the parallelism (Schwartz (1968) calls it properly quasi-parallelism) of the space axes is retained.

Let us now generalise this procedure to six dimensions. After the first part of the derivation we are left with two frames with mutually parallel space axes and mutually parallel time axes. The relative velocity lies in a common space axis and the time unit vector lies in a common time axis. Then the two subsequent rotations of one frame with respect to the other are applied, first in space and later on in time. Thus, a general

0305-4470/81/110433+03$01.50

transformation is obtained in six dimensions between quasi-parallel inertial frames. The corresponding matrix has the form ($c = 1$)

$$L = \begin{pmatrix} A & P \\ Q & R \end{pmatrix}$$

$$\begin{aligned} A_{ij} &= \delta_{ij} + (\gamma - 1)v_i v_j / v^2 \qquad & P_{ij} &= -\gamma \alpha_j v_i \\ R_{ij} &= \delta_{ij} + (\gamma - 1)\alpha_i \alpha_j \qquad & Q_{ij} &= -\gamma \alpha_i v_j \end{aligned} \tag{1}$$

$$i, j = 1, 2, 3 \qquad \gamma = (1 - v^2)^{-1/2} \qquad v^2 = \sum_i v_i^2 \qquad \sum_i \alpha_i^2 = 1$$

v_i are the components of the velocity of origin O' as measured in frame S and α_i are the components of the time unit vector of origin O' as measured in frame S.

The matrix (1) leads to the well known results of special relativity if all time displacements are parallel, e.g. for $\alpha_1 = 1$, $\alpha_2 = \alpha_3 = 0$ and six-vectors of the type $(x_1, x_2, x_3, t_1, t_2 = 0, t_3 = 0)$. It is a solution of Cole's (1980) equations for $\alpha_{0'} = \alpha_0 = \alpha$. Owing to the reciprocity condition it is $\alpha_{0'} = \alpha'_0$ and $\alpha_0 = \alpha'_{0'}$. However, the results obtained with (1) do not agree with well known results of special relativity if all time displacements are not parallel. Let us take a six-vector $(0, 0, 0, t, 0, 0)$ and transform it according to (1) with $v_i = (v, 0, 0)$ and $\alpha_i = (\alpha_{1'2}, 0) = (\cos\phi, \sin\phi, 0)$. A straightforward calculation gives

$$t' = (t_{1'}^2 + t_{2'}^2 + t_{3'}^2)^{1/2} = \gamma t(1 - v^2 \sin^2\phi)^{1/2}. \tag{2a}$$

The proper time t corresponds to the proper decay time τ of an unstable particle and the coordinate time t' to the decay time in flight τ'. The equation of special relativity

$$\tau' = \gamma\tau \tag{2b}$$

is recovered for $\phi = 0$ only.

From measurement with fast muons (Bailey *et al* 1977) follows a limitation for ϕ: $\phi < 0.06 \approx 4°$. This limitation concerning the isotropy of time is not a severe one. Limitations obtained through measurements of the Doppler effect and especially dynamical limitations corresponding to forbidden decays of stable particles are much more severe.

The six-dimensional transformation (1) raises non-trivial questions. Is Cole's scheme capable of giving, for unparallel time displacements, a general transformation in explicit form? Does this transformation comply with the principles of homogeneity and isotropy of space and time (isotropy of time included) and the principle of relativity (reciprocity of inertial frame included)? Does it give results consistent with experimental evidence? It may be possible that in deriving this transformation one has to abandon the reciprocity condition which is well established in four-dimensional special relativity or the principle of isotropy of time which is an extension of the principle of isotropy of space in a symmetrical space–time. Cole's scheme may prove viable, but only after these questions are clarified. As it is obvious that a counterexample cannot replace a rigorous proof, further study is appropriate.

References

Bailey *et al* 1977 Nature **268** 301–5

Cole E A B 1980 *J. Phys. A: Math. Gen.* **13** 109–15
——1981 *J. Phys. A: Math. Gen.* **14** 1821–2
Pars L A 1921 *Phil. Mag.* **42** 249–58
Schwartz H M 1968 *Introduction to Special Relativity* (New York: McGraw-Hill) p 52
Strnad J 1980 *J. Phys. A: Math. Gen.* **13** L389–91